# Melon's Puzzle Packs Volume IV: Fillomino-fillia 2 

By mathgrant and MellowMelon<br>http://mathgrant.blogspot.com, http://mellowmelon.wordpress.com

November 12, 2012

## TABLE OF CONTENTS

| Fillomino Tutorial | $: 2$ |
| :--- | ---: |
| Classic Fillomino (1-9) | $: 4$ |
| Memories of FF1 (10-19) | $: 7$ |
| Snake Fillomino (20-23) | $: 10$ |
| No-rectangles Fillomino (24-27) | $: 11$ |
| Walls Fillomino (28-31) | $: 12$ |
| Nonconsecutive Fillomino (32-35) : 13 |  |
| Liar Fillomino (36-39) | $: 14$ |
| Skyscrapers Fillomino (40-43) | $: 15$ |
| More FF2 Rejects (44-47) | $: 16$ |
| FF3 Teasers? (48-51) | $: 17$ |
| Chimera Fillominos (52-56) | $: 19$ |
| Hints | $: 22$ |
| Solutions | $: 25$ |

## Introduction

This pack contains 56 Fillomino puzzles written by mathgrant and MellowMelon, 26 of which were either appeared in the LMI test Fillomino-fillia 2 or in its preview series. Most of the other puzzles in the pack were considered for the test and rejected. See the following URL for more information about this test:
http://logicmastersindia.com/lmitests/?test=FF2
Each puzzle has a certain number of icons at the top right indicating the difficulty as estimated by the authors. If the icon is a collar, the puzzle was written by mathgrant. If it is a melon, the puzzle was written by MellowMelon. The more icons, the harder the puzzle. But keep in mind that your experience may vary.

If you are struggling with a puzzle, the Hints section provides a tip for each puzzle that may help, usually describing how to get past a sticking point near the start. Solutions are in the back if you get really stumped or want to check your work.

## Fillomino Tutorial

In Fillomino, the goal is to divide all of the grid squares into polyominoes, subject to the following constraints:

1. Every number in the grid must be contained in a polyomino containing that quantity of squares.
2. No two polyominoes containing the same quantity of squares may share an edge.
3. A polyomino may contain one, more than one, or none of the numbers originally given.


To the left is an example Fillomino puzzle. Let's go over how we would solve it.

The first thing to do in many puzzles is find numbers which are greater than 1 and almost trapped. Often the edge is a good place to look for them. Here, the 7 on R1C1, the 2 on R2C1, and the 7 on R4C5 have only one way to expand. So we extend them outward, until either we've made a polyomino of the correct size (as with the 2 ) or until we start having multiple ways to go (as with the 7 s )


Notice that the 7s in the top and the 7s on the bottom are almost running into each other in the center. If we were to put a 7 in R3C4 or R4C3, this could create a contiguous group of eight 7 s . Rule 2 tells us all of them are part of the same polyomino, and this contradicts rule 1 because the polyomino already has more than seven squares. Thus, the two 7-polyominoes in the top and bottom cannot merge. We can mark some walls showing that R3C4 and R4C3 cannot have 7s.


Due to the walls we drew, we can now extend the bottom 7-polyomino a little more until it hits R5C2.

Suppose R2C4 did not have a 7. Then the top 7-polyomino cannot expand to fill 7 spaces without running into the bottom one. We already know this is not allowed. Therefore, the top 7-polyomino must extend to R2C4.

Now the 3 on R3C5 is almost running out of space. It has to use R2C5 if it is going to fill three spaces. Writing this in also seals in the top 7-polyomino, so we know it's 7th square must be R3C2 now.


Now that we know the top 7-polyomino has gone into R3C2, the bottom 7-polyomino cannot use the touching space R4C2. This leaves it with just barely enough room to occupy 7 spaces.

What if the 3-polyomino did not extend into R1C5? Then R1C5 would be completely sealed off from all other squares and thus would be a 1-polyomino. But then it would touch the 1 on R1C4. This is not allowed by rule 2 . Therefore, the 3-polyomino's last space must be R1C5.

All that remains are spaces without numbers. These must be polyominoes for which no number has been given, often called implied polyominoes. R3C4 is now trapped, so it must be a 1 . R4C2 and R4C3 can either be a 2 -polyomino or split into two 1 s . But rule 2 does not allow two 1 s to touch, so it must be a 2 -polyomino. This completes the puzzle.


## Classic Fillomino

The following 9 Fillomino puzzles use the standard set of rules. See page 2 for instructions and an example.

Throughout the pack, puzzles that were part of the FF2 test or preview series will have the eight answer extraction circles provided. If you are not familiar with the test, these circles can be ignored.

IV. 2 FF2: Classic 1

国

IV. 3 FF2: Classic 2

IV. 4



000
IV. 5 FF2: Classic Preview国


IV. 8 FF2: Classic 4


|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | . 4 | 4 | 4 |  | 2 | 1 | 3 |  |  | 6 | 3 |  | 2 |  | 5 | 5 |  |  |
|  |  |  |  |  |  |  |  | 6 | 6 |  |  |  |  |  |  |  |  |  |
| 1 |  | 1 |  | 1 |  | 4 |  |  | 7 |  |  | 3 | 3 | 4 |  | 7 |  | 7 |
| 3 |  | 2 |  | 4 |  | 5 |  |  |  |  |  | 7 | 7 | 4 |  | 7 |  | 7 |
| 3 |  | 3 |  | 6 |  | 5 |  |  |  |  |  | 3 |  | 6 |  | 2 |  | 7 |
| 8 |  | 8 |  | 8 |  | 7 |  | 7 | 7 |  |  | 4 | 4 | 1 |  | 2 |  | 4 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8 | 1 | 2 | 4 |  | 7 | 77 | 7 | 1 |  | 7 | 1 | 4 | 45 |  | 3 | 1 | 7 | 4 |
|  |  |  |  |  |  |  |  |  | 7 |  |  |  |  |  |  |  |  |  |
| 8 |  | 8 |  | 5 |  | 3 |  | 3 | 37 |  |  | 6 | 6 | 8 |  | 9 |  | 7 |
| 8 |  | 8 |  | 8 | 3 | 3 |  |  |  |  |  | 6 | 6 | 9 |  | 9 |  | 3 |
| 8 |  | 8 |  | 8 | 2 | 2 |  |  |  |  |  | 6 | 6 | 9 |  | 9 |  | 3 |
| 8 |  | 8 |  | 7 |  | 2 |  | 3 |  |  |  | 3 | 3 | 9 |  | 9 |  | 3 |
|  |  |  |  |  |  |  |  |  | 5 |  |  |  |  |  |  |  |  |  |
| 7 | 2 | 7 |  |  | 3 | 3 | 3 | 2 |  | 5 | 3 | 3 | 1 |  | 2 | 2 | 1 | 7 |
|  |  |  |  |  |  |  |  |  | 5 |  |  |  |  |  |  |  |  |  |
| 7 |  | 7 |  | 3 |  | 3 |  |  | 2 |  |  | 2 | 2 | 5 |  | 5 |  | 7 |
| 3 |  | 5 |  | 3 |  | 1 |  |  |  |  |  | 1 | 1 | 5 |  | 5 |  | 3 |
| 4 |  | 10 |  | 3 |  | 3 |  |  |  |  |  | 2 | 2 | 4 |  | 4 |  | 4 |
| 10 |  | 4 |  | 10 |  | 1 |  |  | 5 |  |  | 3 | 3 | 5 |  | 5 |  | 7 |
|  |  |  |  |  |  |  |  | 5 | 5 |  |  |  |  |  |  |  |  |  |
|  | - 3 | 3 | 3 |  | 87 |  |  | 6 |  |  | 2 |  |  |  |  | 6 | 1 | 6 |
|  |  |  |  |  |  |  |  |  | 2 |  |  |  |  |  |  |  |  |  |
| 2 |  | 8 |  | 8 |  | 6 |  |  |  |  |  | 5 | 5 | 3 |  | 5 |  | 6 |
| 9 |  | 7 |  | 7 |  | 6 |  |  |  |  |  | 3 | 3 | 3 |  | 5 |  | 6 |
| 7 |  | 7 |  | 7 |  | 3 |  |  |  |  |  | 3 |  | 2 |  | 5 |  | 3 |
| 1 |  | 7 |  | 7 |  | 1 |  |  |  |  |  | 5 | 5 | 2 |  | 5 |  | 3 |
|  |  |  |  |  |  |  |  |  | 25 |  |  |  |  |  |  |  |  |  |
| 1 | 6 | 6 | 6 |  | 24 | 44 | 4 | 1 |  | 9 | 2 | 6 | [ 3 |  | 2 | 1 | 5 | 7 |
|  |  |  |  |  |  |  |  | 9 |  |  |  |  |  |  |  |  |  |  |
| 1 |  | 1 |  | 1 |  | 4 |  |  |  |  |  | 9 |  | 4 |  | 5 |  | 7 |
| 3 |  | 3 |  | 7 |  | 5 |  |  |  |  |  | 6 |  | 4 |  | 5 |  |  |
| 2 |  | 6 |  | 7 |  | 5 |  |  |  |  |  | 2 |  | 2 |  | 7 |  | 5 |
| 2 |  | 6 |  | 1 |  | 1 |  |  |  |  |  | 7 |  | 7 |  | 1 |  | 5 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 6 | 2 |  |  |  | 34 |  |  |  |  | 7 |  |  |  | 1 | 4 | 5 |  |

## Memories of FF1

While Fillomino-fillia 2 revisited two variations of Fillomino-fillia 1, this section will revisit all seven. Here are the relevant rules; a label by each puzzle tells which variation applies.

Cipher: In addition to the usual rules, the given numbers have been replaced by letters. All instances of a particular letter represent the same number, but two different letters must represent different numbers.

Sum: In addition to the usual rules, the grid contains some cages. The number at the top left of each cage gives the sum of all numbers that appear inside of it. Numbers may be repeated in cages.

Shikaku: In addition to the usual rules, every polyomino must be shaped like a rectangle.

Greater-than: In addition to the usual rules, the grid will contain inequality signs. Each sign must point from a larger polyomino to a smaller one.

Star: In addition to the usual rules, not all of the cells will be contained in polyominoes; the remaining cells will contain stars. Every row and every column must contain two stars, and no two stars may be in cells which share a corner or an edge.

Shape: In addition to the usual rules, the shapes shown beside the puzzle must appear as polyominoes in the grid. Shapes may be rotated, but not reflected.

Even-odd: In addition to the usual rules, the odd numbers must form a single polyomino, and the even numbers must similarly form a single polyomino.

IV. 11 (Sum) 気


IV. 14 (Greater-than)


IV. 15 FF2: Sum Preview 萬


OO


OO


IV. 17 FF2: Sum

IV. 19 (Even-odd)


## Snake Fillomino

A snake with a head and tail and unknown length must be drawn in the grid so that it contains all shaded cells. The snake does not touch itself, even at a point. The remaining spaces must be divided into polyominoes satisfying the usual Fillomino rules. The snake may
 touch polyominoes of the same size as itself.

IV. 21 FF2: Top Snake

IV. 23 FF2: Bottom Snake


0000
0000

## No-rectangles Fillomino

In addition to the usual rules, none of the polyominoes can form a rectangle.

IV. 24

IV. 25 FF2: Top No-rect 萬

IV. 27 FF2: Bottom No-rect


00000000

## Walls Fillomino

In addition to the usual rules, a pair of cells with a thick border between them must contain different numbers.

| 1 |  | 6 |
| :---: | :---: | :---: |
|  |  | - |
| 1 | 4 | 6 |
|  | 1 | 6 |
| 2 | 4 | 6 |
|  | 6 | 6 |
| 2 | 4 | 4 |$\quad$| 4 | 2 | 2 |
| :---: | :---: | :---: |
| 4 | 4 | 4 |


IV. 29 FF2: Top Walls

IV. 30 FF2: Walls Preview 囲



0000

IV. 31 FF2: Bottom Walls


## Nonconsecutive Fillomino

In addition to the usual rules, two orthogonally adjacent cells may not contain consecutive numbers.

|  | 5 | 2 |
| :---: | :---: | :---: |
| 4 |  |  |
|  | 3 |  |
| 7 | 7 |  |


| 4 | 1 | 5 | 5 | 2 |
| :---: | :---: | :---: | :---: | :---: |
| 4 | 4 | 1 | 5 | 2 |
| 4 | 1 | 3 | - | 7 |
| 1 | 3 | 3 | 5 | 7 |
| 7 | 7 | 7 | 7 | 7 |


IV. 33 FF2: Top Noncons. 貍

IV. 34 FF2: Noncons. Preview


IV. 35 FF2: Bottom Noncons.


## Liar Fillomino

In addition to the usual rules, exactly one given number in each row and column is false.


| v.36 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 |  |  |  |  |  |  |
| 4 | 6 | 3 | 3 |  |  | 2 |
|  |  | 2 |  |  | 5 |  |
|  | 2 | 2 | 96 | 1 | 5 |  |
|  | 2 |  |  | 4 |  |  |
| 2 |  |  | 3 | 3 | 6 | 2 |
| 1 | 2 |  |  |  |  | 2 |


IV. 38 FF2: Liar Preview

IV. 39 FF2: Bottom Liar


## Skyscrapers Fillomino

In addition to the usual rules, the numbers in the grid should be treated as building heights. Numbers on the outside of the grid tell how many buildings are visible when looking from that direction. A building obscures all buildings behind it whose height is equal to or smaller than itself.

IV. 42 FF2: Sky Preview 萬




## More FF2 Rejects

These are additional puzzles that did not make it into FF2 or its preview series. The puzzles are labelled with the appropriate variation; see one of the previous pages for rules and an example.

IV. 45 (No-rect)

|  |  | 3 |
| :--- | :--- | :--- |
| 6 | 6 | 6 |
|  | 9 | 5 |
| 6 | 6 | 5 |
| 6 | 6 | 5 |
| 6 | 6 | 5 |
| 6 | 6 | 5 |
|  | 5 | 5 |
| 11 | 11 | 5 |

IV. 46 (No-rect)

| 5 |  | 5 |  | 4 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 4 |  |  | 7 |  | 5 |  |
| 5 |  |  |  |  | 4 |  |  | 7 |  |
|  | 4 |  | 4 |  |  | 5 |  | 5 |  |
| 5 |  |  |  | 5 |  |  |  |  |  |
|  |  |  |  |  | 4 |  |  | 7 |  |
|  | 3 |  | 3 |  |  | 4 |  | 5 |  |
| 3 |  |  |  | 3 |  |  |  | 5 |  |
|  | 3 |  | 3 |  |  | 5 |  |  |  |
|  |  |  |  |  | 5 |  | 5 | 5 |  |

IV. 47 (Snake)


## FF3 Teasers?

While we cannot positively confirm that a third iteration of Fillomino-fillia will be on the way next year, we're interested in the possibility. Here are some variations you might see when the time comes.

LITS Fillomino: The rule that no two polyominoes of the same size may share an edge is specifically waived for tetrominoes, which must follow the rules of LITS. (All tetrominoes must be connected through their edges, there may not be $2 x 2$ areas, and there may not be tetrominoes of the same shape sharing an edge.) The number 4 never appears as a given in the grid, and is instead replaced by the letters L, I, T, and S , indicating the shape of the tetromino.
IV. 48

IV. 49

톱․


Sentry Fillomino: If two equal numbers are in the same row or column, they must be part of the same polyomino.
(Originally done by Anderson Wang: http://qzqxq.wordpress.com/)

IV. 51


## Chimera Fillominos

These final puzzles combine some of the variations seen elsewhere in the pack.

Nonconsecutive
No-rect
Fillomino: Two orthogonally adjacent cells may not contain consecutive numbers, and no polyomino may be shaped like a rectangle.

Nonconsecutive
IV. 52

困

IV. 53


Touchy Shikaku Fillomino: Every region must touch (by side) at least one region with a number of cells one more or one less than itself, and all polyominoes must be shaped like a rectangle.
IV. 54

IV. 55



Potpourri Fillomino 2: This puzzle combines five variations from Fillomino-fillia 2 , as described below.

Snake: A snake with a head and tail and unknown length must be drawn in the grid so that it contains all shaded cells. The snake does not touch itself, even at a point. The remaining spaces must be divided into polyominoes satisfying the usual Fillomino rules in addition to those below.

No-rectangles: None of the polyominoes can form a rectangle.
Walls: A pair of cells with a thick border between them must not contain two identical numbers. At most one cell of the pair can be part of the snake.

Nonconsecutive: Two orthogonally adjacent cells may not contain consecutive numbers.

Skyscrapers: The numbers in the grid should be treated as building heights. Numbers on the outside of the grid tell how many buildings are visible when looking from that direction. A building obscures all buildings behind it whose height is equal to or smaller than itself. The snake is completely invisible to the Skyscraper clues; the first building seen in a row or column is the first non-snake cell.


## Hints

IV.1: Look for numbers with just enough space for their polyomino, and fill that space up.
IV.2: Make sure you avoid combining polyominoes if it would make them too large.
IV.3: The only implied polyominoes in this puzzle are 2s. Look out for unreachable squares next to a 1 s , among other things.
IV.4: This puzzle is about finding polyominoes with limited space to grow and extending them. Start from the right and work your way left.
IV.5: The logic in heavy use here is a surrounded space next to a 1 . One of the touching numbers has to fill it.
IV.6: When considering how to complete a polyomino, be very wary of sealing off some squares. There may not be an implied polyomino that can fill them.
IV.7: The theme of this puzzle is a pair of numbers and only two ways out for both. Regardless of which number goes which way, no other number is allowed to block them.
IV.8: The 3 s are the key to this puzzle. There is barely enough space on the edge, and only if many of the 3 s extend toward the center.
IV.9: Watch for spaces that touch completed $1 \mathrm{~s}, 2 \mathrm{~s}$, etc. and are thus forced to have large numbers, even if you don't see a given one nearby.
IV.10: You can figure out which letter is 1 from the givens alone, so all other letters are at least 2.
IV.11: A 14 -sum has to have a polyomino of size at least 7 .
IV.12: The center gives you the letters that are 1 and 3 immediately. So all other letters are either 2 or at least 4.
IV.13: Figure out why the 10 can't go up to row 1 . Also, there are only a few ways to pack the $6 s$ in the bottom right
IV.14: Do careful bookkeeping on the three by three boxes. How many spaces need small numbers, and how large can the big polyominoes be as a result?
IV.15: Think about the possible ways the 6 and 9 cages can be resolved. Both the top left and bottom right can be starting points.
IV.16: A two by two area can have at most one star. After the obvious way of partitioning the grid into 25 such two by two areas, you'll find five are covered with givens, while there are 20 stars.
IV.17: A wall can be drawn between any two-cell cage with an odd sum. Remember that small sums effectively block large polyominoes.
IV.18: It's Statue Park all over again! Note that 2 times 30 plus 8 times 5 is the size of the grid. The X, I, and Y pentominoes should be the first to get placed.
IV.19: If two spaces are surrounded by 1 s and evens, you're not putting any odd numbers in them.
IV.20: The snake does not necessarily take the direct path between shaded squares.
IV.21: Make sure to keep polyominoes from sealing the snake off. Also, make sure the snake doesn't isolate any squares next to 1 s .
IV.22: The critical square is R6C4. The bottom left runs out of space if it's not a snake cell.
IV.23: The center configuration can only work if two of those shaded squares are the head and tail of the snake.
IV.24: All 3s have to be L-triominoes in this variation.
IV.25: The puzzle will deviously attempt to get you very close to making rectangles with many of the polyominoes, some quite large. Watch out for these.
IV.26: Think a little about the possible shapes a 4 can have in this variation before starting. And be especially careful of sealing off areas that are smaller than 3 cells.
IV.27: There are only two ways to do the 3 s in the top and bottom rows. And if the 6 s in corners aren't going to be rectangles, they have to escape out somewhat far.
IV.28: Be especially careful of implied polyominoes in this one, and keep track of which spaces the 1 s touch.
IV.29: This puzzle is all about making sure the 6 s on the inside and outside don't touch.
IV.30: There is a lot of long distance interaction in this puzzle. If you have trouble with a quadrant, make sure you haven't forgotten the numbers on the other side of the walls.
IV.31: The only possibilities for the nonsquare tetrominoes are 4, 1-3, 3-1, and 1-2-1. For squares, they have to be 4 or 3-1. Try to find pairs of touching squares where one must be 4 and the other must be 3-1.
IV.32: This puzzle can be straightforwardly done if you start in the top left and go clockwise.
IV.33: Be careful not to seal in areas that are too small if that would make them consecutive with nearby polyominoes.
IV.34: This puzzle is about polyominoes having just enough space to fill. This will be harder to find than for the classic variation, so analyze the possible area a polyomino can fill carefully.
IV.35: Obviously most of the solution will be filled by 10 s and 11 s . Be extremely careful of what goes in between. Don't trap 1s next to other 1s.
IV.36: You can easily find all seven liars without writing a single number.
IV.37: The rows and columns on the edge have only one number, so it must be a liar. This allows a lot of standard Fillomino steps in the center.
IV.38: If you have six 5 s in a row, one has to lie. Be wary of the tricky configuration of 5 s coming into play in the middle of the solve as well, and make sure you don't form an "L" that's too long.
IV.39: What happens if a line of four 7s in the center is all true? The bottom right is the starting point once you find some liars in the corners.
IV.40: Row 3 is the starting point; both clues can be replaced entirely with inequalities
in the grid. In the middle, the critical skyscraper clues are in row 4 and column 2.
IV.41: Keep very good track of what the maximum number can be in a row/column. Most of the logic is driven by remembering this.
IV.42: There is a very large polyomino in this puzzle. Be wary of making assumptions about upper limits on how big the numbers can be.
IV.43: A short hint can't do much; the logic is long, involved, and varied. Start with the high numbers, and keep track of the known maximums in each row and column carefully.
IV.44: Use the three 1s and 2s to find the starting liars; the clue layout alone will give you many of the others.
IV.45: The center is an extremely tight fit when taken together with the variation's constraint. For example, what happens if no pair of 5 s in columns 6-7 merge?
IV.46: By now you've learned about not making implied 1 s and 2 s in this variation. In this puzzle, you need to avoid implied polyominoes that are larger but still rectangular. The bottom left is the start.
IV.47: What number can R1C8 have?
IV.48: There are certain regions of the puzzle through which the LITS pieces can't connect, although it won't be obvious at first glance.
IV.49: A wide variety of constraints on R3C2 force it to have a surprisingly large number. Also, what happens if the 3 on R3C4 is not allowed to use the space to its right?
IV.50: The two 18 s can't be the same polyomino. You need to find some rectangular shapes with perimeter at most 20 that they can fit in.
IV.51: There are 7s all over the grid, and their touchy polyominoes are relatively large. There's barely enough room for them all.
IV.52: As usual, the key is to avoid hidden polyominoes that are so small they'd be rectangular. Here, you also need to make sure they are not consecutive with nearby polyominoes.
IV.53: An isolated 3 by 4 rectangular area needs at least one polyomino of size at least 6 in these rules.
IV.54: There are very few ways to have a polyomino be touchy with the 8 or 10 .
IV.55: Focus on trying to organize all the 6 s in the top to start. Make sure you don't seal off any 3 s if there isn't a 4 nearby.
IV.56: The starting point simultaneously uses the Walls, No-rectangle, and Skyscraper rules, and it takes place at the walls touching the corners.

## IV. 1

| 4 | 4 | 4 | 5 | 5 | 5 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 6 | 6 | 6 | 5 | 2 | 2 |
| 6 | 6 | 1 | 2 | 2 | 4 | 5 |
| 6 | 2 | 2 | 4 | 4 | 4 | 5 |
| 1 | 3 | 5 | 2 | 2 | 1 | 5 |
| 2 | 3 | 5 | 1 | 3 | 3 | 5 |
| 2 | 3 | 5 | 5 | 5 | 3 | 5 |

IV. 4

| $\left.\frac{3}{3} \right\rvert\,$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  | 4 |  | 2 |  |  |  |  |
|  | 5 |  | 5 |  |  | 2 |  |  |  |
|  |  |  | 6 |  |  | 3 | 6 |  |  |
|  |  |  | 7 |  |  |  |  |  |  |
|  |  |  |  | 5 |  | 5 |  |  |  |
|  | , |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

IV. 7

## IV. 8




Solutions

IV. 5


## IV. 3


IV. 6

| 3 | 2 | 5 | 4 | 4 | 3 | 3 | 4 | 4 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 2 | 5 | 4 | 2 | 2 | 3 | 4 | 2 | 2 |
| 3 | 5 | 5 | 4 | 3 | 3 | 2 | 2 | 4 | 1 |
| 4 | 5 | 1 | 5 | 4 | 3 | 4 | 4 | 4 | 2 |
| 4 | 2 | 4 | 5 | 4 | 5 | 5 | 5 | 5 | 2 |
| 4 | 2 | 4 | 5 | 4 | 5 | 9 | 9 | 2 | 9 |
| 4 | 1 | 4 | 5 | 4 | 3 | 9 | 1 | 2 | 2 |
| 6 | 2 | 4 | 5 | 3 | 3 | 9 | 9 | 9 | 9 |
| 6 | 2 | 1 | 2 | 2 | 1 | 4 | 4 | 4 | 2 |
| 6 | 6 | 6 | 6 | 3 | 3 | 3 | 4 | 1 | 2 |

IV. 9


## IV. 10

| Z ORR | R O | Z $\mathrm{R}^{\text {R }}$ |
| :---: | :---: | :---: |
| 0 O | OO | R |
| R:R:R | Z R | Z |
| 0 O | R R | A |
| R:R:R | OR | A- ${ }^{\text {A }}$ |
| $\bigcirc \mathrm{O}$ | OR |  |
| OR R |  | A Z |

IV. 13

|  |  | 9 | 9 |  |  | 25 |  | 3 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 9 | 9 | 9 | 1 | 6 | 6 |  |  | 1 |
| 8 | 9 | 9 | 9 | 3 | 6 | 5 |  | 0:10 | 6 |
| 8 | 6 | 6 | 2 | 3 | 6 | 65 |  | $0: 10$ | 6 |
| 8 | 6 | 6 | 2 | 3 | 6 | 5 |  | 010 | 6 |
| 8 | 6 | 66 | 5 | 4 | 6 | 62 |  | $0: 10$ | 6 |
| 8 | 4 | 4 | 5 | 4 | 6 | -2 |  | 0:10 | 6 |
| 8 |  | 44 | 5 | 4 | 8 | 8 |  | 8 | 6 |
| 6 | 6 | 6 | 5 | 4 |  | 8 |  | 8 |  |
|  | 6 |  |  |  | 6 | 6 |  | 6 |  |

IV. 16


## IV. 19

| 6 | 6 | 6 | 6 | 6 | 4 | 6 | 6 | 6 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 7 | 7 | 1 | 6 | 4 | 1 | 5 | 1 | 6 |
| 7 | 4 | 7 | 7 | 4 | 4 | 5 | 5 | 5 | 6 |
| 7 | 4 | 4 | 1 | 5 | 5 | 1 | 5 | 1 | 4 |
| 3 | 3 | 4 | 2 | 2 | 5 | 5 | 2 | 4 | 4 |
| 4 | 3 | 2 | 4 | 4 | 2 | 5 | 2 | 4 | 2 |
| 4 | 1 | 2 | 1 | 4 | 2 | 1 | 3 | 1 | 2 |
| 4 | 3 | 3 | 3 | 4 | 8 | 8 | 3 | 4 | 4 |
| 4 | 1 | 4 | 1 | 8 | 8 | 1 | 3 | 1 | 4 |
| 2 | 2 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 4 |

IV. 11

IV. 14

IV. 17

IV. 20

IV. 12

| 2 | 3 | 3 | 6 | 6 | 4 | 3 | 3 | $(3)$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 4 | 3 | 6 | 6 | 4 | 1 | 2 | 2 | 4 |
| 6 | 4 | 4 | 3 | 6 | 4 | 2 | 3 | 4 | 4 |
| 6 | 4 | 3 | 3 | 6 | 4 | 2 | 3 | 3 | 4 |
| 6 | 6 | 6 | 6 | 1 | 3 | 6 | 6 | 6 | 3 |
| 7 | 4 | 4 | 4 | 3 | 3 | 1 | 6 | 6 | 3 |
| 7 | 2 | 4 | 2 | 6 | 1 | 7 | 6 | 1 | 3 |
| 7 | 2 | 3 | 2 | 6 | 3 | 7 | 7 | 7 | 7 |
| 7 | 7 | 3 | 3 | 6 | 3 | 4 | 2 | 2 | 7 |
| 7 | 7 | 6 | 6 | 6 | 3 | 4 | 4 | 4 | 7 |

IV. 15


## IV. 18

| 5 | 5 | 30 | 5 | 5 | 5 | 5 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## IV. 21


IV. 22

IV. 25

IV. 28

IV. 31

IV. 23

IV. 26

| 16 | 4 | 4 | 4 | 6 | 6 | 4 | 4 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 4 | $16: 16$ | 4 | 6 | 4 | 6 | 6 | 6 |  |
| 16 | $16: 16$ | 4 | 4 | 6 | 6 | 4 | 6 | 4 |  |
| 16 | 4 | $16: 16$ | 4 | 6 | 4 | 4 | 6 | 4 |  |
| 16 | 4 | $16: 16: 16: 16$ | 4 | 7 | 4 | 4 |  |  |  |
| 4 | 4 | 16 | 4 | 4 | 4 | 7 | 7 | 7 | 7 |
| 7 | 7 | 4 | 5 | 5 | 4 | 7 | 4 | 4 | 7 |
| 4 | 7 | 4 | 4 | 5 | 5 | 4 | 4 | 5 | 4 |
| 4 | 7 | 7 | 4 | 5 | 4 | 5 | 5 | 5 | 4 |
| 4 | 4 | 7 | 7 | 4 | 4 | 4 | 5 | 4 | 4 |


\section*{IV. 29 <br> | 2 | 6 | 6 | 6 | 6 | 3 | 6 | 6 | 6 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 6 | 6 | 1 | 3 | 3 | 6 | 8 | 8 | 6 |
| 6 | 2 | 1 | 6 | 6 | 6 | 1 | 6 | 8 | 8 |
| 6 | 2 | 6 | 6 | 2 | 3 | 6 | 6 | 8 | 6 |
| 6 | 6 | 1 | 6 | 2 | 3 | 3 | 6 | 8 | 6 |
| 4 | 6 | 2 | 2 | 6 | 2 | 2 | 6 | 8 | 6 |
| 4 | 6 | 1 | 6 | 6 | 6 | 5 | 6 | 8 | 6 |
| 4 | 4 | 6 | 6 | 5 | 5 | 5 | 5 | 6 | 6 |
| 2 | 2 | 4 | 1 | 6 | 6 | 6 | 6 | 3 | 3 |
| 4 | 4 | 4 | 6 | 6 | 3 | 3 | 3 | 1 | 3 |}

IV. 32

IV. 24

| 3 | 3 | 5 | 6 | 6 | 6 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 3 | 5 | 5 | 5 | 6 | 6 |
| 4 | 4 | 3 | 5 | 4 | 5 | 5 |
| 4 | 3 | 3 | 4 | 4 | 7 | 5 |
| 6 | 6 | 6 | 3 | 4 | 7 | 5 |
| 6 | 3 | 6 | 3 | 3 | 7 | 5 |
| 6 | 3 | 3 | 7 | 7 | 7 | 7 |

IV. 27

IV. 30

IV. 33
$\left.\begin{array}{|c|c:c|c|c:c|c:c:c|}\hline 1 & 3 & 3 & 8 & 2 & 2 & 4 & 4 & 4 \\ \hline 4 & 1 & 3 & 8 & 8 & 8 & 8 & 8 & 8 \\ \hdashline 4 & 4 & 7 & 3 & 3 & 3 & 6 & 6 & 3 \\ \hdashline 4 & 7 & 7 & 7 & 7 & 7 & 1 & 6 & 10 \\ \hline 2 & 5 & 7 & 4 & 4 & 4 & 9 & 6 & 10 \\ \hline 2 & 5 & 5 & 1 & 4 & 9 & 9 & 6 & 6 \\ \hline 4 & 2 & 5 & 3 & 9 & 9 & 10 & 4 & 4\end{array}\right)$

## IV. 34


IV. 37

IV. 40


## IV. 43


IV. 35

IV. 38

| 2 | 2 | 2 | 1 | 2 | 4 | 4 | 4 | 3 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 3 | 2 | 5 | 2 | 4 | 3 | 3 | 1 | 3 |
| 4 | 3 | 5 | 5 | 5 | 5 | 5 | 5 | 3 | 1 |
| 4 | 4 | 5 | 1 | 9 | 9 | 9 | 5 | 3 | 3 |
| 2 | 1 | 5 | 9 | 1 | 9 | 1 | 5 | 9 | 3 |
| 2 | 4 | 5 | 2 | 9 | 9 | 5 | 5 | 9 | 4 |
| 4 | 4 | 5 | 9 | 1 | 5 | 9 | 5 | 9 | 4 |
| 7 | 4 | 5 | 5 | 5 | 5 | 5 | 5 | 9 | 4 |
| 7 | 7 | 7 | 7 | 7 | 2 | 3 | 3 | 9 | 1 |
| 1 | 4 | 4 | 4 | 4 | 2 | 2 | 3 | 2 | 9 |

## IV. 41


IV. 44


## IV. 36

|  |  | 4.4 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
|  |  | 62 |  |  |  |  |
|  |  | 22 | 96 |  |  |  |
| $\overline{2}$ |  | 214 | 4 |  |  | 5 |
| 2 |  | 3.3 |  |  |  |  |
|  |  | 2 |  |  |  |  |

IV. 39

| 3 | 1 | 2 | 3 | 3 | 3 | 8 | 8 | 8 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 3 | 2 | 8 | 8 | 8 | 8 | 8 | 2 | 3 |
| 1 | 3 | 7 | 7 | 7 | 4 | 4 | 4 | 3 | 3 |
| 2 | 4 | 7 | 1 | 7 | 7 | 1 | 5 | 5 | 5 |
| 2 | 4 | 7 | 7 | 7 | 7 | 7 | 5 | 1 | 5 |
| 6 | 4 | 5 | 7 | 7 | 7 | 7 | 4 | 2 | 2 |
| 6 | 5 | 5 | 5 | 7 | 7 | 4 | 4 | 5 | 5 |
| 6 | 4 | 4 | 4 | 4 | 5 | 7 | 4 | 5 | 5 |
| 6 | 1 | 1 | 5 | 5 | 5 | 7 | 4 | 5 | 5 |
| 2 | 6 | 3 | 3 | 3 | 7 | 7 | 7 | 7 | 7 |

IV. 42

IV. 45

| 6 | 9 | 9 | 9 | 9 | 3 | 4 | 3 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 6 | 9 | 6 | 6 | 3 | 3 | 4 | 4 |
| 3 |  |  |  |  |  |  |  |  |
| 6 | 9 | 9 | 9 | 6 | 5 | 5 | 4 | 5 |
| 6 | 6 | 7 | 6 | 6 | 6 | 5 | 7 | 7 |
| 7 | 7 | 7 | 7 | 7 | 5 | 5 | 7 | 5 |
| 6 | 6 | 7 | 6 | 6 | 6 | 7 | 7 | 7 |
| 6 | 8 | 8 | 8 | 6 | 5 | 5 | 4 | 5 |
| 6 | 6 | 8 | 6 | 6 | 5 | 4 | 4 | 4 |
| 6 | 8 | 8 | 8 | 8 | 5 | 5 | 11 | 5 |
| $11: 11: 11: 11: 11: 11: 11: 1: 11: 11$ |  |  |  |  |  |  |  |  |

## IV. 46


IV. 49

IV. 52


## IV. 55


IV. 47

IV. 50

IV. 53


## IV. 56


IV. 48

IV. 51

IV. 54

| 2 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 2 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1 | 3 | 3 | 3 | 1 | 2 | 2 | 4 | 4 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 4 | 4 |
| 2 | 6 | 7 | 3 | 1 | 5 | 5 | 5 | 5 | 5 |
| 2 | 6 | - | 3 | 2 | 2 | 1 | 2 | 2 | 3 |
| 3 | 6 | 7 | 3 | 9 | 9 | 9 | 10 | 10 | 3 |
| 3 | 6 | 7 | 1 | 9 | 9 | 9 | 10 | 10 | 3 |
| 3 | 6 | 7 | 2 | 9 | 9 | 9 | 10 | 10 | 2 |
| 2 | 6 | 7 | 2 | 1 | 2 | 2 | 10 | 10 | 2 |
| 2 | 1 | 7 | 1 | 3 | 3 | 3 | 10 | 10 | 1 |

